

# Flux-flow in $d$ -wave superconductors: Low temperature universality and scaling

N.B. Kopnin<sup>1,2</sup> and G.E. Volovik<sup>1,3</sup>

<sup>1</sup>*Landau Institute for Theoretical Physics, 117334 Moscow, Russia*

<sup>2</sup>*Laboratoire de Physique des Solides, Université Paris-Sud, Bât 510, 91405 Orsay, France*

<sup>3</sup>*Helsinki University of Technology, Low Temperature Laboratory, P. O. Box 2200, FIN-02015 HUT, Finland*

(February 1, 2008)

We demonstrate that superclean  $d$ -wave superconductors with  $\Delta_\infty^2 \tau / E_F \gg 1$  display a novel type of vortex dynamics: At low temperatures, both dissipative and transverse components of the flux-flow conductivity are found to approach universal values even in the limit of infinite relaxation time. A finite dissipation in the superclean limit is explained in terms of the Landau damping on zero-frequency vortex modes which appear due to minigap nodes in the bound-state spectrum in the vortex core. In the moderately clean regime the scaling law at low  $T$  and low field is obtained.

PACS numbers: 74.25.Fy, 74.25.Jb, 74.72.-h, 74.60.Ge

The Landau description of low-temperature thermodynamics and low-frequency dynamics of Fermi and Bose liquids is based on the concept of predominating low-energy quasiparticles. In the same way, the low-frequency dynamics of vortices in Fermi superfluids and superconductors appears to be also determined by low-energy excitations. For Fermi superfluids and superconductors with a gap in the spectrum, the relevant low-energy quasiparticles are concentrated in vortex cores. The anomalous spectral branch of quasiparticles localized in the core, which crosses zero as a function of the orbital quantum number, was first found by Caroli, de Gennes and Matricon [1] for the simplest case of axisymmetric vortices in  $s$ -wave superconductors. Effects of these quasiparticles on the vortex dynamics were considered in a number of papers (see, for example, [2,3]).

The  $d$ -wave superconductivity adds new aspects to this problem: due to the gap nodes, gapless quasiparticles exist also outside vortex cores. The low-energy fermions which are far from vortex cores but still are under the influence of the vortex superflow velocity (i) introduce non-analytical terms in the free energy of the mixed state of a  $d$ -wave superconductor [4]; (ii) result in a singularity in the vortex density of states [5] and (iii) lead to a peculiar scaling law for thermodynamic and kinetic properties of  $d$ -wave superconductors at low  $T \ll T_c$  in small magnetic fields  $B \ll H_{c2}$  [5–7]. Here we show that the vortex dynamics is also influenced by these far-distant excitations bound to vortices. This results in a new scaling law for the Ohmic and Hall conductivities in the mixed state of a  $d$ -wave superconductor. Moreover, a novel effect appears: the vortex motion is accompanied by the Landau damping on vortex modes giving rise to a finite dissipation even in the superclean limit.

It is known [8,9] that, in a  $d$ -wave superconductor, impurity scattering broadens the gap nodes to the impurity band whose width is  $\delta \sim T_c \exp(-\pi \Delta_0 \tau)$  in the Born approximation. For temperatures below the impurity bandwidth  $\delta$ , the current response function in

the Meissner state,  $\mathbf{j} = -Q(\omega)\mathbf{A}(\omega)$  displays unusual behavior: The “conductivity”  $\sigma(0)$  defined as  $Q(\omega \rightarrow 0) = n_s e^2 / mc - i(\omega/c)\sigma(0)$ , saturates to a scattering-independent universal value [9,10] corresponding to the Drude conductivity with the effective relaxation time  $1/\tau \sim T_c$ . In the present Letter, we demonstrate that a similar effect exists in the mixed state of  $d$ -wave superconductors: Both longitudinal (Ohmic) and transverse (Hall) components of the conductivity tensor approach finite universal values even in the limit of infinite relaxation time, if temperatures and magnetic fields are low enough. This universal behavior of the flux-flow conductivity is realized in superclean systems with the mean free path  $\ell \gg (E_F/T_c)\xi$ . We trace the finite dissipation in the superclean limit back to the resonant absorption (Landau damping) on zero-frequency vortex modes which appear due to the nodes in the minigap  $\omega_0$ , i.e., in the distance between bound-state levels in the vortex core. Note that the width of the abovementioned impurity band  $\delta$  is vanishingly small for superconductors with an impurity scattering rate satisfying the superclean condition.

We start with the spectrum of relevant low-energy fermionic quasiparticles assuming that  $T \ll T_c$ . Since our results are not sensitive to the details of the quasiparticle spectrum, we take the simplest spectrum, which has the tetragonal symmetry and four nodes. In a bulk  $d$ -wave superconductor,  $E(\mathbf{p}) = \sqrt{\xi_p^2 + \Delta_\infty^2 \sin^2(2\alpha)}$ . Here  $\xi_p = \epsilon(p) - E_F$ . We consider an isotropic Fermi surface around the  $c$  axis and assume that the vortex is axisymmetric. The magnetic field is along the symmetry axis which is the  $z$  axis of the coordinate frame with the positive direction along the vortex circulation  $\hat{\mathbf{z}} = \text{sgn}(e)\mathbf{B}/B$ ;  $\alpha$  is the azimuthal angle of  $\mathbf{p}$  counted from one of the 4 nodes in the gap ( $\alpha = 0$ ).

*Quasiparticles bound to the vortex.* The low-energy spectrum of Caroli-de-Gennes-Matricon quasiparticles around a vortex contains an anomalous branch. Excitations on this branch are characterized by two canoni-

cally conjugated variables, the angle  $\alpha$  and the angular momentum  $Q$ . In the quasiclassical approximation the variables  $\alpha$  and  $Q$  are commuting, and the spectrum is  $E(Q, \alpha) = -\omega_0(\alpha)Q$ . For a  $d$ -wave superconductor close to the gap nodes  $|\sin(2\alpha)| \ll 1$ , the distance between the energy levels, i.e., the minigap is [4,5]

$$\omega_0(\alpha) = \frac{2\Delta_\infty^2 \sin^2(2\alpha)}{v_\perp p_\perp} \ln \frac{1}{|\sin(2\alpha)|}. \quad (1)$$

Here  $p_\perp$ ,  $v_\perp$  are the projections of the Fermi momentum and velocity on the  $x-y$  plane. According to Eq. (1), the minigap  $\omega_0$  as a function of  $\alpha$  has nodes of a higher order than the gap itself. We shall show that the strong singularity in  $\omega_0(\alpha)$  results in a nonvanishing dissipation even in the superclean limit  $\Delta_\infty^2 \tau / E_F \gg 1$  when the quasiclassical minigap  $\omega_0(\alpha)$  is well resolved.

*Kinetic equation* for the distribution function  $f(t, \alpha, Q)$  for a system of fermions characterized by canonically conjugated variables has a conventional form [11]:

$$\frac{\partial f}{\partial t} - \frac{\partial f}{\partial \alpha} \frac{\partial E}{\partial Q} + \frac{\partial E}{\partial \alpha} \frac{\partial f}{\partial Q} = -\frac{f - f_0}{\tau_0}. \quad (2)$$

Here  $\partial E / \partial Q = -\omega_0(\alpha)$ ;  $f_0$  is the equilibrium Fermi distribution; the collision integral is written in a  $\tau$ -approximation. A rigorous consideration shows that the collision integral for localized excitations does not have singularities as a function of  $\alpha$ , thus it can be considered as a constant on order of the relaxation time  $\tau$  in the normal state.

If the vortex moves with a constant velocity  $\mathbf{u} = u(\hat{\mathbf{x}} \cos \phi + \hat{\mathbf{y}} \sin \phi)$  with respect to the lattice, the Doppler shift of the energy  $E \rightarrow E(Q, \alpha) - \mathbf{p}\mathbf{u}$  produces the “driving force”  $(\partial E / \partial \alpha) = up_\perp \sin(\alpha - \phi)$  acting on quasiparticles. The third term in the lhs of kinetic equation (2) contains this force multiplied by  $\partial f_0 / \partial Q = -\omega_0(\alpha)(df_0 / dE)$ . Introducing the longitudinal and transverse response to this perturbation and taking into account that, for the tetragonal symmetry, the response does not depend on the direction of the vortex motion with respect to the crystal lattice, one obtains

$$\delta f = -up_\perp \frac{df_0}{dE} [\gamma_H \cos(\alpha - \phi) + \gamma_O \sin(\alpha - \phi)] \quad (3)$$

where  $\gamma_O(\alpha)$  and  $\gamma_H(\alpha)$  satisfy two coupled first-order differential equations

$$\begin{aligned} \frac{\partial \gamma_O}{\partial \alpha} - \gamma_H - U(\alpha)\gamma_O + 1 &= 0, \\ \frac{\partial \gamma_H}{\partial \alpha} + \gamma_O - U(\alpha)\gamma_H &= 0 \end{aligned} \quad (4)$$

where  $U(\alpha) = [\omega_0(\alpha)\tau]^{-1}$ . These two real functions are combined into one complex function  $\gamma_O = \text{Im } W(\alpha)$ ,  $\gamma_H = \text{Re } W(\alpha)$ , which obeys the equation

$$(\partial W / \partial \alpha) - (U + i)W + i = 0. \quad (5)$$

*The force acting on the vortex* from the environment where it moves is the momentum transferred from the excitations:

$$\mathbf{F}_{\text{env}} = - \int \frac{dp_z}{2\pi} \frac{dQ}{2\pi} \frac{d\alpha}{2\pi} \frac{\partial \mathbf{p}}{\partial t} \delta f \quad (6)$$

Using  $\partial \mathbf{p} / \partial t = -\nabla E = [\hat{\mathbf{z}} \times \mathbf{p}](\partial E / \partial Q)$  we get

$$\begin{aligned} \mathbf{F}_{\text{env}} &= - \int \frac{dp_z}{2\pi} \frac{dQ}{2\pi} \frac{d\alpha}{2\pi} [\hat{\mathbf{z}} \times \mathbf{p}_\perp] \frac{\partial E}{\partial Q} \delta f \\ &= \pi n \langle \gamma_H \rangle_F [\hat{\mathbf{z}} \times \mathbf{u}] - \langle \gamma_O \rangle_F \mathbf{u}. \end{aligned} \quad (7)$$

Here  $n$  is the electron density;  $\langle \dots \rangle_F$  is the average over the whole Fermi surface. The force  $\mathbf{F}_{\text{env}}$  should be balanced by the Lorentz force  $\mathbf{F}_L = \pi n [\mathbf{v}_s \times \hat{\mathbf{z}}]$ . The so called mutual friction parameters  $\langle \gamma_O \rangle_F$  and  $\langle \gamma_H \rangle_F$  are thus coupled to the flux-flow longitudinal and Hall conductivities. At low temperatures,

$$\langle \gamma_O \rangle_F = (B\sigma_O / n|e|c), \quad \langle \gamma_H \rangle_F = (B\sigma_H / nec). \quad (8)$$

If the Fermi surface has electron-like and hole-like pockets, the conductivities at low temperatures are [3]

$$\sigma_O = \frac{|e|c}{B} [n_e \langle \gamma_O \rangle_{F,e} + n_h \langle \gamma_O \rangle_{F,h}], \quad (9)$$

$$\sigma_H = \frac{ec}{B} [n_e \langle \gamma_H \rangle_{F,e} - n_h \langle \gamma_H \rangle_{F,h}]. \quad (10)$$

where  $\langle \dots \rangle_{F,e}$  and  $\langle \dots \rangle_{F,h}$  are the averages over the electron-like and hole-like parts of the Fermi surface, respectively;  $n_e$  and  $n_h$  are the numbers of electrons and holes.

*Solution of the kinetic equation* (5) is

$$W = e^{[i\alpha + F(\alpha)]} \left( C - i \int_0^\alpha e^{-[i\alpha' + F(\alpha')]} d\alpha' \right) \quad (11)$$

where  $F(\alpha) = \int_0^\alpha U(\alpha') d\alpha'$ . In the moderately clean limit,  $\Delta_0^2 \tau / E_F \ll 1$ , the potential  $U(\alpha)$  is always large, and we obtain the local solution as in an  $s$ -wave superconductor [3]

$$\gamma_0(\alpha) = \omega_0(\alpha)\tau, \quad \gamma_H(\alpha) = [\omega_0(\alpha)\tau]^2. \quad (12)$$

The conductivities are

$$\sigma_O \sim \frac{nec}{B} \frac{\Delta_\infty^2 \tau}{E_F} \ln \frac{T_c}{T} \quad (13)$$

and  $\sigma_H / \sigma_O \sim (\Delta_\infty^2 \tau / E_F) \ln(T_c / T)$  which are similar to the results for an  $s$ -wave superconductor [2,3,12].

In the superclean limit,  $\Delta_0^2 \tau / E_F \gg 1$ , the situation is more interesting. The potential  $U(\alpha)$  is small almost everywhere except for vicinities of the gap nodes where it diverges:  $U(\alpha) \sim 1 / [\Gamma(\delta\alpha)^2]$  at  $|\delta\alpha| \ll 1$ . Here  $\Gamma \sim \Delta_0^2 \tau / E_F$ . However there is a physical infrared cutoff  $\alpha_{\min}$  for the angle below which the divergent potential  $U(\alpha)$  saturates. For example, if the vortices are separated

by a finite distance  $R \sim \xi \sqrt{H_{c2}/B}$  with  $B \ll H_{c2}$ , i.e.,  $R \gg \xi$ , the Doppler shift of the energy  $\mathbf{p}_F \mathbf{v}_s \sim T_c(\xi/R)$  (where  $\mathbf{v}_s$  is the vortex-induced superflow velocity) affects the spectrum Eq. (1) when the excitation energy becomes comparable with the gap  $\Delta_0 |\sin(2\alpha)|$ . This determines the cutoff  $\alpha_{min} = \sqrt{B/H_{c2}}$  [4,5]. In addition, a finite temperature introduces a cutoff  $\alpha_{min} = T/T_c$  since an excitation with a smaller  $\alpha$  is already above the gap  $\Delta_0 |\sin(2\alpha)|$ . The relaxation rate  $1/\tau$  also provides a cutoff in the form  $\alpha_{min} = 1/\tau T_c$ . Since the parameter  $1/\tau T_c$  is much smaller than all the other cutoff parameters in our case, we can write in general

$$\alpha_{min} = \max\left(T/T_c, \sqrt{B/H_{c2}}\right). \quad (14)$$

Assuming that  $U(\alpha < \alpha_{min}) = 1/(\Gamma \alpha_{min}^2)$  one can insert the truncated potential into Eq. (11). Since the potential  $U(\alpha)$  is concentrated near the nodes one can neglect the potential almost in the whole range of  $\alpha$  outside a small vicinity of the nodes. Eq. (11) has the form  $W = 1 + Ae^{i\alpha}$  where the complex constant  $A$  can be found by matching with the solution in the vicinity of the node,  $\alpha \ll 1$ , i.e.,  $W(\alpha) \propto e^{F(\alpha)}$ . This provides the boundary condition  $W(+0) = e^{2\lambda} W(-0)$  across the node at  $\alpha = 0$ . Here

$$2\lambda = F(+0) - F(-0) \sim 1/(\Gamma \alpha_{min}). \quad (15)$$

The response function has the same tetragonal symmetry as the underlying system,  $W(\alpha + \pi/2) = W(\alpha)$ . Together with the above boundary condition, this gives

$$W(\alpha) = 1 - \frac{1 - e^{2\lambda}}{1 - ie^{2\lambda}} e^{i\alpha}, \quad 0 < \alpha < \pi/2, \quad (16)$$

which is to be periodically continued to the rest of angles with the period  $\pi/2$ . We have

$$\langle \gamma_H \rangle = 1 - \frac{4}{\pi} \frac{1}{1 + \coth^2 \lambda}, \quad \langle \gamma_O \rangle = \frac{4}{\pi} \frac{\coth \lambda}{1 + \coth^2 \lambda}. \quad (17)$$

Here  $\langle \dots \rangle$  is the average over the azimuthal angle  $\alpha$ .

*Magnetic field and temperature dependence.* In the low-field limit  $B/H_{c2} \ll (T/T_c)^2$  we have  $\lambda = T_c/(T\Gamma)$ . At not very low temperatures  $1/\Gamma \ll T/T_c \ll 1$ , the parameters in Eq. (17) are

$$\langle \gamma_O \rangle \sim T_c/(T\Gamma), \quad 1 - \langle \gamma_H \rangle \sim [T_c/(T\Gamma)]^2. \quad (18)$$

For  $T \sim T_c$ , these expressions transform into usual solutions for a superclean *s*-wave superconductor [2,3]. For very low temperatures  $T/T_c \ll 1/\Gamma$ , the parameters become independent of the scattering time

$$\langle \gamma_O \rangle = 2/\pi, \quad \langle \gamma_H \rangle = 1 - (2/\pi). \quad (19)$$

For fields  $B/H_{c2} \gg (T/T_c)^2$ , we have  $\lambda = (1/\Gamma) \sqrt{H_{c2}/B}$ . The asymptotics for  $1/\Gamma^2 \ll B/H_{c2}$  are

$$\langle \gamma_O \rangle \sim (1/\Gamma) \sqrt{H_{c2}/B}, \quad 1 - \langle \gamma_H \rangle \sim H_{c2}/(\Gamma^2 B). \quad (20)$$

Eq. (20) suggests the magnetic-field dependences of the flux-flow conductivity in the form  $\sigma_O \propto B^{-3/2}$ . In the limit  $B/H_{c2} \ll 1/\Gamma^2$  we recover Eq. (19).

*Universal superclean limit.* For  $\Gamma \gg 1$  and for low temperatures and fields such that

$$T/T_c \text{ and } \sqrt{B/H_{c2}} \ll 1/\Gamma, \quad (21)$$

the longitudinal and transverse conductivities approach finite universal values independent of the scattering mean free time  $\tau$ :

$$\sigma_O = \frac{(n_e + n_h)|e|c}{B} \langle \gamma_O \rangle; \quad \sigma_H = \frac{(n_e - n_h)ec}{B} \langle \gamma_H \rangle \quad (22)$$

where  $\langle \gamma_O \rangle$  and  $\langle \gamma_H \rangle$  are given by Eq. (19). The scattering rate affects only the range of temperatures and fields, Eq. (21), where this asymptotic behavior is established. With lowering the temperature for fixed  $\Gamma \gg 1$  and  $B$ , one expects an increase in  $\sigma_O$  and, finally, a crossover to the regime where both longitudinal and Hall conductivities have universal values independent of the mean free path. Together with the conductivities, the Hall angle reaches its universal value determined only by the numbers of electron and holes:  $\tan \theta_H = (\pi/2 - 1)(n_e - n_h)/(n_e + n_h)$ . The universal value of  $|\tan \theta_H|$  is expected to be a local minimum at low temperatures due to an increasing  $\sigma_O$ .

Experimentally, the sample purity needed for observation of the universal behavior of Eq. (21) is rather high; it requires  $\ell/\xi_0 \gg 10$  to 100 depending on the ratio  $E_F/T_c$  for the particular high- $T_c$  superconductor. Nevertheless, there are indications that such a regime can be realized in practice: the superclean limit was nearly approached in Ref. [13] with  $\Gamma \sim 1$  for a presumably *d*-wave 60 K YBaCuO compound [14].

*Landau damping.* To understand the reason for a finite dissipation in the limit of an infinite relaxation time  $\tau = \infty$ , let us consider the  $\omega$ -dependence of the response function  $W$ . It is obtained by the substitution  $i/\tau \rightarrow \omega$ . Therefore,  $\lambda \rightarrow -i\pi\omega/4E_0$  where

$$E_0 = \left[ \frac{1}{2\pi} \int_0^{2\pi} \frac{d\alpha}{\omega_0(\alpha)} \right]^{-1}. \quad (23)$$

We have

$$\langle \gamma_H(\omega) \rangle + i \langle \gamma_O(\omega) \rangle = 1 + \frac{2(1+i)}{\pi} \frac{1 - \exp(-i\pi\omega/2E_0)}{1 - i \exp(-i\pi\omega/2E_0)}.$$

This response function has poles at  $\omega_k = (4k+1)E_0$  where  $k$  is an integer. Note that  $E_0$  in Eq. (23) is the true interlevel distance in the spectrum of bound states. It can be obtained by quantization of the azimuthal motion using the Bohr-Zommerfeld rule [5]:  $\oint Q(\alpha) d\alpha = 2\pi(m + \gamma)$ , where  $m$  is an integer and  $\gamma \sim 1$ . Since

$Q(\alpha) = -E/\omega_0(\alpha)$ , the quantum-mechanical spectrum is  $E = -(m + \gamma)E_0$ , assuming that the integral in Eq. (23) converges due to a finite magnetic field. The poles correspond to eigenmodes in the quasiparticle distribution when the frequency matches the resonance transition between the levels with different  $m$ . In  $s$ -wave superconductors, the selection rule for transitions caused by motion of an axisymmetric vortex is  $\Delta m = \pm 1$  with the resonance at  $\omega = E_0$ . In  $d$ -wave superconductors other harmonics are mixed due to the tetragonal symmetry giving rise to resonances at  $\omega = \omega_k$ .

When  $\alpha_{\min} \rightarrow 0$ , thus  $E_0 \rightarrow 0$ , the poles densely fill the  $\omega$ -axis. This leads to a finite density of states and thus to a finite Ohmic resistivity at  $\omega = 0$ : the parameter

$$\langle \gamma_O \rangle = \frac{2}{\pi} \lim_{E_0 \rightarrow 0} \text{Im} \left[ \frac{1+i}{\Omega} \int_0^\Omega d\omega \frac{1 - e^{-i\pi\omega/2E_0}}{1 - ie^{-i\pi\omega/2E_0}} \right]$$

goes to  $2/\pi$  as  $\Omega \rightarrow 0$ . The attenuation in the zero-frequency limit is a special type of the Landau damping of vortex motion due to the nodes in the minigap.

The true interlevel distance  $E_0$  introduces a new energy scale which does not exist for superconductors without gap nodes. For conventional superconductors, there is a single characteristic value, the minigap  $\omega_0$ , which marks a crossover in the relaxation rate  $1/\tau$  between two regimes: a dissipative viscous flux flow for  $\omega_0\tau \ll 1$ , and a dissipationless vortex motion for  $\omega_0\tau \gg 1$ . For superconductors with gap nodes, there are two critical values: the average quasiclassical interlevel distance  $\langle \omega_0 \rangle$  and the true interlevel distance  $E_0$  which can be less than  $\langle \omega_0 \rangle$ . Indeed, in the superclean limit  $\langle \omega_0 \rangle \tau \gg 1$ , the true minigap is  $E_0 \sim \langle \omega_0 \rangle \alpha_{\min}$ ; near the nodes, the quasiclassical levels approach each other on much shorter distances which can only be resolved by quantization of the angular motion.

The condition of Eq. (21) means that  $E_0\tau \ll 1$ . A finite density of states appears when the true quantum-mechanical interlevel distance is smaller than the level width. Therefore one has three regimes: (i) Moderately clean regime  $\langle \omega_0 \rangle \tau \ll 1$  with a high dissipation, (ii) intermediate universal regime  $E_0\tau \ll 1$  with a finite dissipation at zero  $\omega$ , where the quasiclassical levels are well resolved on average but the true minigap is smaller than the scattering rate; and (iii) the extreme superclean case  $E_0\tau \gg 1$  when the true levels are well separated and thus there is no dissipation.

The regime with an universal flux-flow conductivity is reminiscent of the asymptotic behavior of the current response function in the Meissner state. In the flux-flow state, however, the regime of the universal conductivity is reached at higher temperatures,  $T \ll T_c/\Gamma$ . The reason is that the minigap in the spectrum of low-energy excitation which are responsible for the vortex dynamics, has nodes of a higher order than the gap itself. Therefore, the effect of the nodes in the flux-flow regime is more pronounced. The universal values of the conduc-

tivities correspond to an effective scattering rate  $1/\tau$  of the order of the characteristic minigap  $\omega_0$ . This is also in parallel with the behavior of the response function in the Meissner state, where the effective scattering rate is of the order of the characteristic gap  $\Delta_\infty$  [9].

*In conclusion*, we have calculated the flux-flow dynamics in  $d$ -wave superconductors at low temperatures. In the moderately clean case,  $\Delta_\infty^2 \tau / E_F \ll 1$ , the Ohmic and Hall components of the flux-flow conductivity are similar to those for conventional superconductors. However, for superclean systems with  $\Delta_\infty^2 \tau / E_F \gg 1$ , the magnetic field dependence is unusual. Moreover, at low  $B$  and  $T$ , both Ohmic and Hall components approach universal values independent of relaxation. A finite dissipation in the superclean limit is explained in terms of the Landau damping on zero-frequency vortex modes which appear due to minigap nodes in the spectrum of bound states in the vortex core. Since  $\Delta_\infty/E_F$  is not very small in high- $T_c$  compounds, the superclean regime is accessible in those high purity crystals, where the scaling law for the specific heat has been verified [15].

This work was supported by the Russian Foundation for Fundamental Research grant No. 96-02-16072, by the RAS program ‘‘Statistical Physics’’, and partially by Swiss National Foundation cooperation grant No. 7SUP J048531.

- 
- [1] C. Caroli, P. G. de Gennes, and J. Matricon, Phys. Lett. **9**, 307 (1964).
  - [2] N. B. Kopnin and V. E. Kravtsov, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 631 (1976) [JETP Lett. **23**, 578 (1976)].
  - [3] N. B. Kopnin, A. V. Lopatin, Phys. Rev. B **51**, 15291 (1995).
  - [4] G.E. Volovik, Pis'ma ZhETF, **58**, 457 (1993); [JETP Lett., **58**, 469 (1993)].
  - [5] N.B. Kopnin and G.E. Volovik, Pis'ma ZhETF, **64**, 641 (1996); [JETP Lett., **64**, 690 (1996)].
  - [6] S.H. Simon, P.A. Lee, Phys. Rev. Lett., **78**, 1548 (1997).
  - [7] G.E. Volovik, Pis'ma ZhETF, **65**, 465 (1997).
  - [8] L.P. Gorkov and P.A. Kalugin, Pis'ma ZhETF **41**, 208 (1985) [JETP Lett. **41** 253 (1985)].
  - [9] P. A. Lee, Phys. Rev. Lett. **71**, 1887 (1993).
  - [10] Y. Sun and K. Maki, Europhys. Lett. **32**, 355 (1995).
  - [11] M.Stone, Phys. Rev. B **54**, 13222 (1996).
  - [12] A. I. Larkin and Yu. N. Ovchinnikov, Pis'ma ZhETF **23**, 210 (1976) [JETP Lett. **23** 187 (1976)].
  - [13] J. M. Harris, Y. F. Yan, O. K. C. Tsui, Y. Matsuda, and N. P. Ong, Phys. Rev. Lett. **73**, 1711 (1994).
  - [14] D. A. Brawner and H. R. Ott, Phys. Rev. B **53**, 8249 (1996).
  - [15] A. Junod, M. Roulin, B. Revaz, *et al*, in M2S-HTSC-V - Proceedings (Beijing, China, Feb. 28 - Mar. 4, 1997), Ed. Yu-Sheng He, to appear in Physica C.